

Regularized Structural Equation Modeling with Both Factors and Components

Gyeongcheol Cho¹, Ji Yeh Choi², Marko Sarstedt^{3,4}, Heungsun Hwang^{5,6}

1. Ohio State University, Columbus, USA

2. York University, Toronto, Canada

3. Ludwig-Maximilians-University Munich, Germany

4. Babeş-Bolyai University, Cluj-Napoca, Romania

5. McGill University, Montreal, Canada

6. Sogang University, Seoul, Korea

Author Note

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Correspondence concerning this article should be addressed to Heungsun Hwang, Department of Psychology, McGill University, 2001 McGill College Avenue, Montreal, QC H3A 1G1, Canada. Email: heungsun.hwang@mcgill.ca.

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Abstract

Integrated generalized structured component analysis (IGSCA) was recently developed to estimate structural equation models that include both factors and components. In this paper, we propose integrating regularization into IGSCA to address the issues of multicollinearity and variable selection. The proposed method, termed regularized IGSCA, enables the imposition of L_1 (ridge) or L_2 (lasso) penalty on loadings and path coefficients separately. We seek to minimize a penalized least squares objective function for parameter estimation. We conduct a simulation study to evaluate the performance of the method and apply it to real data to illustrate its empirical usefulness.

Keywords: Structural equation modeling with factors and components; integrated generalized structured component analysis; regularization; multicollinearity; variable selection

1. Introduction

Structural equation modeling (SEM) is widely used to statistically examine theory-driven relationships between constructs, such as job satisfaction, socioeconomic status, work stress, etc. Constructs refer to abstract conceptual variables, and they are statistically represented as entities linked to empirical data or observed variables, thereby allowing researchers to test hypotheses about the relationships between them (Rigdon, 2012).

In SEM, constructs have been statistically represented as (common) factors or components, giving rise to two SEM domains: factor-based vs. component-based (e.g., Cho et al., 2022; Jöreskog & Wold, 1982; Rigdon, 2012; Rigdon et al., 2017). In factor-based SEM, all constructs are represented as factors, also often referred to as latent variables (e.g., Rhemtulla et al., 2020), which are to capture the common variance shared by observed variables under the assumption that a construct is considered an external reality independent of observed variables and the cause of their covariance (Borsboom et al., 2003). Observed variables whose covariance pattern is governed by factors are called reflective or effect indicators, which are substantially intercorrelated (Bollen, 1989, p. 65; Edwards & Bagozzi, 2000; MacCallum & Browne, 1993). Factors have been used to represent various psychological constructs in cognition, emotion, personality, and mental disorders (Borsboom et al., 2003).

Conversely, in component-based SEM, constructs are represented as weighted composites or components of observed variables under the assumption that observed variables constitute different facets or aspects of a broader construct and combining them forms this construct (e.g., Cho et al., 2022). Thus, components are determined by their variables rather than leading to the variables' covariance pattern. Observed variables forming components are called composite indicators (Bollen, 2011; Bollen & Bauldry, 2011; Grace & Bollen, 2008). Unlike reflective

indicators, composite indicators are not necessarily highly intercorrelated because no common causes underlie them.

Accordingly, when deciding whether a factor or component is more appropriate to represent a given construct, a primary consideration is whether this construct causes or precedes its indicators, or is determined or preceded by them (Borsboom et al., 2003; Hwang, Sarstedt, et al., 2023). For example, in the classical *g* theory of intelligence (e.g., Jensen, 1998), a single construct *g* is assumed to derive observed test items. Then, a factor may represent intelligence (van der Maas et al., 2014). Conversely, in psychopathology, body composition can be represented as a component as it is an aggregation of several measures such as body fat, muscle, and bone mineral (Borga et al., 2018). Similarly, in neuroimaging, a region of interest (ROI) can be represented as a component, which is a cluster of brain voxels (Hwang, Cho, Jin, et al., 2021; Jung et al., 2012; Zhou et al., 2016).

Over the past decades, the two SEM domains have remained largely distinct from each other in that researchers have applied either factor- or component-based SEM, thereby representing all the constructs in their models as factors or components (Rigdon et al., 2017). However, as many sciences, including psychology, are increasingly becoming interdisciplinary (Box-Steffensmeier et al., 2022), researchers often need to consider different types of constructs simultaneously, some of which can be better represented as factors while others as components (e.g., Hwang, Cho, Jung, et al., 2021). Figure 1 illustrates such a model, where socioeconomic status indirectly influences job satisfaction through work stress as well as directly affects job satisfaction (e.g., Blustein, 2006; Brown & Lent, 2005; Penney & Spector, 2005). When educational attainment, income, and occupational prestige are employed to measure socioeconomic status, the construct is a component and not a factor (American Psychological

Association, 2007). Similarly, work stress can be represented as a component as the psychological demands of workload, workers' perceived sense of control over their performance, safety stressors, work organization, and work atmosphere all contribute to work stress (Aittomäki et al., 2003). Conversely, job satisfaction can be represented as a factor underlying several indicators concerning overall and specific aspects of job satisfaction (Macdonald & MacIntyre, 1997; Oshagbemi, 1999). Furthermore, in the model, it is hypothesized that the magnitude and/or direction of the mediated effect of socioeconomic status on job satisfaction via work stress may vary across different levels of resilience, which has been represented as a factor (Connor & Davidson, 2003). Note that Figure 1 only shows the structural model with the components and factors.

===== Insert Figure 1 around here =====

Existing methods for factor- and component-based SEM are not well-suited for estimating models with both factors and components. It is well-known that a component-based SEM method, such as partial least squares path modeling or PLS-SEM (Lohmöller, 1989; Wold, 1973, 1982) or generalized structured component analysis (GSCA; Hwang & Takane, 2004, 2014), will produce positively biased estimates of factor loadings for reflective indicators and negatively biased—attenuated—estimates of path coefficients connecting factors only (e.g., Dijkstra, 2010; Hwang et al., 2010; Velicer & Jackson, 1990). Conversely, a factor-based SEM method will provide negatively biased estimates of component loadings for composite indicators and positively biased estimates of path coefficients connecting components only (e.g., Cho et al., 2022; Rhemtulla et al., 2020). Furthermore, Hwang et al. (2021) have shown that a component-based SEM will likely yield negatively biased estimates of path coefficients relating factors to components, whereas a factor-based SEM tends to provide positively biased estimates of the

same path coefficients. Therefore, traditional methods for each SEM domain would always result in biased estimates of several parameters in the model containing both factors and components.

To address this issue, integrated generalized structured component analysis (IGSCA; Hwang, Cho, Jung, et al., 2021) was developed that estimates models with factors and components simultaneously. IGSCA combines two SEM methods, GSCA and GSCA with measurement errors incorporated (GSCA_M; Hwang et al., 2017). GSCA is a well-established method for component-based SEM, whereas GSCA_M is a data-matrix-based approach to factor-based SEM. By integrating these methods into a single framework, IGSCA can appropriately estimate models with factors and components, providing unbiased estimates of all parameters in the models. It is also robust to the occurrence of improper solutions. This method has been implemented into the free software GSCA Pro (Hwang et al., 2024; Hwang, Cho, et al., 2023).

In this paper, we propose incorporating regularization into IGSCA to address potential multicollinearity or perform variable selection. Multicollinearity is prevalent in SEM (Hwang, 2009; Hwang & Takane, 2014, Chapter 8; Marsh et al., 2004), making it difficult to interpret the unique influence of one variable on another. Moreover, it tends to inflate the variances of parameter estimates, leading to Type II errors. This inflation can also make the mean square errors of the estimates larger, indicating that the estimates become farther away from their parameters on average. Ridge or L_2 regularization (Hoerl & Kennard, 1970) is widely used to address multicollinearity. It tends to produce biased estimates by shrinking them toward zero yet can substantially reduce their variances. Consequently, on average, the ridge estimates are typically closer to their parameters than their ordinary least squares counterparts. This tendency becomes more evident in small samples and/or when multicollinearity is present (e.g., Takane & Hwang, 2007).

Variable selection, or parsimonious model selection, is also important in SEM (e.g., Baumann et al., 2002; Jacobucci et al., 2016; Miller, 2002). When a factor or component is associated with too many indicators in the measurement model, there is a high chance of including irrelevant indicators, which can result in poor fit. Moreover, removing irrelevant factors or components from the structural model can help reveal the most relevant structures of the phenomena being studied (e.g., Baumann et al., 2002). Such a step can also aid in specifying a more parsimonious model that reduces the risk of overfitting and may excel in terms of predictive power (Hastie et al., 2009, p. 38). Lasso or L_1 regularization (Tibshirani, 1996) is a popular approach for variable selection. It imposes a shrinkage constraint on parameter estimates similar to L_2 regularization, but it also tends to force some estimates to be exactly zero, automatically selecting a subset of variables (Berk, 2008, p. 64).

The proposed method, termed regularized IGSCA, aims to regularize loadings and path coefficients. This method estimates these parameters by minimizing a penalized least squares objective function that includes both L_1 and L_2 regularizations. It allows for the imposition of penalties on loadings and path coefficients separately. While there are regularized methods for factor-based SEM (e.g., Huang et al., 2017; Jacobucci et al., 2016; Jung & Park, 2018; Yuan & Lin, 2006) and for component-based SEM (e.g., Hwang, 2009; Tenenhaus & Tenenhaus, 2011), these methods are only suitable for addressing multicollinearity or variable selection in models with either factors or components alone. Consequently, they are not appropriate for handling these issues in models with both factors and components.

The paper is organized as follows. In Section 2, we begin by reviewing IGSCA briefly and then present the technical underpinnings of the proposed method. In Section 3, we conduct a simulation study to evaluate the performance of regularized IGSCA, focusing on how well this

method performs in parameter recovery. In Section 4, we apply regularized IGSCA to real data to demonstrate its usefulness. In the final section, we summarize the implications of the proposed method and discuss directions for future research.

2. Method

2.1. Integrated Generalized Structured Component Analysis (IGSCA)

In this section, we present a description of IGSCA, which serves as the foundation for the proposed method, with a focus on its model specification and parameter estimation.

The majority of the content is adapted from the original papers on IGSCA (Hwang, Cho, Jung, et al., 2021) and GSCA_M (Hwang et al., 2017).

As with GSCA and GSCA_M, IGSCA involves three sub-models: measurement, structural, and weighted relation models. The measurement model specifies the relationships between indicators and factors/components, whereas the structural model expresses the relationships between factors/components. Let $\mathbf{z}_1$ and $\mathbf{z}_2$ denote J_1 by one and J_2 by one vectors of composite and effect indicators, respectively. Let $\boldsymbol{\gamma}_1$ and $\boldsymbol{\gamma}_2$ denote P_1 by one and P_2 by one vectors of components and factors, respectively. We assume that all indicators, components, and factors are standardized to have zero means and unit variances. Let $\mathbf{C}_1$ and $\mathbf{C}_2$ denote P_1 by J_1 and P_2 by J_2 matrices of loadings relating $\boldsymbol{\gamma}_1$ and $\boldsymbol{\gamma}_2$ to $\mathbf{z}_1$ and $\mathbf{z}_2$, respectively. Let $\mathbf{u}$ denote a J_2 by one vector of unique variables. Let $\mathbf{D}$ denote a J_2 by J_2 diagonal matrix of unique loadings.

The measurement model for IGSCA is a combination of those for GSCA and GSCA_M. The measurement model for GSCA is given as

$$\mathbf{z}_1 = \mathbf{C}_1' \boldsymbol{\gamma}_1 + \boldsymbol{\varepsilon}_1, \quad (1)$$

where $\boldsymbol{\varepsilon}_1$ is the portion of $\mathbf{z}_1$ left unexplained by γ_1 , as in linear regression (Hwang & Takane, 2014, Chapter 2). The measurement model for GSCA_M is given as

$$\mathbf{z}_2 = \mathbf{C}_2'\boldsymbol{\gamma}_2 + \mathbf{D}\mathbf{u} + \boldsymbol{\varepsilon}_2, \quad (2)$$

where $\mathbf{C}_2'\boldsymbol{\gamma}_2$ and $\mathbf{D}\mathbf{u}$ indicate common and unique parts of $\mathbf{z}_2$, respectively, and $\boldsymbol{\varepsilon}_2$ is the portion of $\mathbf{z}_2$ left unexplained by their common and unique parts (Hwang et al., 2017). If the common factor analytic model also holds for a sample, $\boldsymbol{\varepsilon}_2$ will be zero (Velicer & Jackson, 1990). It is assumed that $\boldsymbol{\gamma}_2$ is uncorrelated with $\mathbf{u}$, i.e., $\boldsymbol{\gamma}_2\mathbf{u}' = \mathbf{u}\boldsymbol{\gamma}_2' = \mathbf{0}$, and $\mathbf{u}$ is column-wise orthonormalized, i.e., $\mathbf{u}\mathbf{u}' = \mathbf{I}$, where $\mathbf{I}$ is an identity matrix of an appropriate order.

The measurement model for IGSCA is then given as

$$\begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{C}_1' & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_2' \end{bmatrix} \begin{bmatrix} \boldsymbol{\gamma}_1 \\ \boldsymbol{\gamma}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{D}\mathbf{u} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\varepsilon}_1 \\ \boldsymbol{\varepsilon}_2 \end{bmatrix}$$

$$\mathbf{z} = \mathbf{C}'\boldsymbol{\gamma} + \mathbf{s} + \boldsymbol{\varepsilon}, \quad (3)$$

where $\mathbf{z} = [\mathbf{z}_1; \mathbf{z}_2]$, $\mathbf{C} = \begin{bmatrix} \mathbf{C}_1' & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_2' \end{bmatrix}$, $\boldsymbol{\gamma} = [\boldsymbol{\gamma}_1; \boldsymbol{\gamma}_2]$, $\mathbf{s} = [\mathbf{0}; \mathbf{D}\mathbf{u}]$, and $\boldsymbol{\varepsilon} = [\boldsymbol{\varepsilon}_1; \boldsymbol{\varepsilon}_2]$.

Similarly, the weighted relation model for IGSCA combines those for GSCA and GSCA_M. The weighted relation model for GSCA is given as

$$\boldsymbol{\gamma}_1 = \mathbf{W}_1'\mathbf{z}_1, \quad (4)$$

where $\mathbf{W}_1$ is a J_1 by P_1 matrix of weights assigned to $\mathbf{z}_1$ (Hwang & Takane, 2014, Chapter 2).

The weighted relation model for GSCA_M is given as

$$\boldsymbol{\gamma}_2 = \mathbf{W}_2'(\mathbf{z}_2 - \mathbf{D}\mathbf{u}), \quad (5)$$

where $\mathbf{W}_2$ is a J_2 by P_2 matrix of weights assigned to $\mathbf{z}_2$, whose unique parts are removed (Hwang et al., 2017). This way of expressing factors dates back to Guttman (1944), which has based many well-known factor-analytic methods, such as the diagonal, centroid, and principal axes methods (refer to Mulaik, 2009, Chapter 7).

The weighted relation model for IGSCA is then expressed as

$$\begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} \mathbf{W}_1' & \mathbf{0} \\ \mathbf{0} & \mathbf{W}_2' \end{bmatrix} \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 - \mathbf{D}\mathbf{u} \end{bmatrix}$$

$$\boldsymbol{\gamma} = \mathbf{W}'\mathbf{z}^*, \quad (6)$$

where $\mathbf{z}^* = [\mathbf{z}_1; \mathbf{z}_2 - \mathbf{D}\mathbf{u}]$ and $\mathbf{W} = \begin{bmatrix} \mathbf{W}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{W}_2 \end{bmatrix}$.

Let $\mathbf{B}$ denote a P by P matrix of path coefficients relating $\boldsymbol{\gamma}$ among themselves, where $P = P_1 + P_2$. The structural model for IGSCA is expressed as

$$\boldsymbol{\gamma} = \mathbf{B}'\boldsymbol{\gamma} + \boldsymbol{\zeta}, \quad (7)$$

where $\boldsymbol{\zeta}$ is the error term for $\boldsymbol{\gamma}$. The $\mathbf{B}$ matrix contains path coefficients relating components to factors, as well as those among either factors or components only.

IGSCA integrates the sub-models into the following unified formulation.

$$\begin{bmatrix} \mathbf{z} \\ \boldsymbol{\gamma} \end{bmatrix} = \begin{bmatrix} \mathbf{C}' \\ \mathbf{B}' \end{bmatrix} \boldsymbol{\gamma} + \begin{bmatrix} \mathbf{s} \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\varepsilon} \\ \boldsymbol{\zeta} \end{bmatrix}$$

$$\boldsymbol{\psi} = \mathbf{A}'\boldsymbol{\gamma} + \mathbf{v} + \mathbf{e}, \quad (8)$$

where $\boldsymbol{\psi} = [\mathbf{z}; \boldsymbol{\gamma}]$, $\mathbf{A}' = [\mathbf{C}'; \mathbf{B}']$, $\mathbf{v} = [\mathbf{s}; \mathbf{0}]$, and $\mathbf{e} = [\boldsymbol{\varepsilon}; \boldsymbol{\zeta}]$.

To estimate all parameters in $\mathbf{W}$, $\mathbf{A}$ and $\mathbf{v}$, IGSCA seeks to minimize the following least squares objective function.

$$\varphi = \sum_{l=1}^N \mathbf{e}_l' \mathbf{e}_l, \quad (9)$$

subject to $\text{diag}(\boldsymbol{\gamma}\boldsymbol{\gamma}') = N\mathbf{I}$, $\boldsymbol{\gamma}_2\mathbf{u}' = \mathbf{u}\boldsymbol{\gamma}_2' = \mathbf{0}$, and $\mathbf{u}\mathbf{u}' = \mathbf{I}$.

To estimate the parameters, IGSCA relies on an algorithm that iteratively updates a set of parameters in the least squares sense with the other sets fixed until no substantial differences in the estimates occur between iterations. This estimation procedure does not require a

distributional assumption on the indicators. IGSCA utilizes the (non-parametric) bootstrap method (Efron, 1979) to estimate the standard errors or confidence intervals of parameter estimates.

2.2. Regularized IGSCA

We now propose integrating L_1 (lasso) and L_2 (ridge) regularization into IGSCA. This method, regularized IGSCA, aims to estimate parameters by minimizing the following penalized least squares objective function

$$\phi = \frac{\tau}{2} \varphi + \lambda_C \mathbf{1}' |\mathbf{C}^{\circ\tau}| \mathbf{1} + \lambda_B \mathbf{1}' |\mathbf{B}^{\circ\tau}| \mathbf{1}, \quad (10)$$

where $\mathbf{1}$ is a vector of ones of an appropriate order, $\mathbf{T}^{\circ\tau}$ indicates the elementwise exponentiation (or Hadamard power) of any matrix $\mathbf{T}$, i.e., $\mathbf{T}^{\circ\tau} = (\mathbf{T}_{ij})^\tau$, and λ_C and λ_B are non-negative tuning parameters for loadings and path coefficients, respectively. In (10), $|\mathbf{T}|$ means the absolute value of $\mathbf{T}$. When $\tau = 1$, the two terms $\mathbf{1}' |\mathbf{C}^{\circ\tau}| \mathbf{1}$ and $\mathbf{1}' |\mathbf{B}^{\circ\tau}| \mathbf{1}$ become L_1 penalties, whereas when $\tau = 2$, they are L_2 penalties. As shown in (10), the method regularizes loadings and path coefficients separately. Parameter estimation draws on an extended version of the IGSCA algorithm, which implements the regularization, also allowing for regularizing only part of the loadings and/or path coefficients.

Let $\mathbf{Z} = [\mathbf{z}_1, \dots, \mathbf{z}_J]$ denote an N by J matrix of effect and composite indicators, where $\mathbf{z}_j$ is an N by 1 vector of the j th indicator ($j = 1, \dots, J$), and $J = J_1 + J_2$. Let $\mathbf{\Gamma} = [\boldsymbol{\gamma}_1, \dots, \boldsymbol{\gamma}_P]$ denote an N by P matrix of components or factors, and $\boldsymbol{\gamma}_p$ is an N by 1 vector of the p th component or factor ($p = 1, \dots, P$). Let $\mathbf{U} = [\mathbf{u}_1, \dots, \mathbf{u}_{J_2}]$ denote an N by J_2 matrix of unique variables, where $\mathbf{u}_j$ is an N by 1 vector of the j th unique variable. Let $\mathbf{S} = [\mathbf{U}\mathbf{D}, \mathbf{0}]$. The objective function (10) can then be re-expressed as

$$\phi = \frac{\tau}{2} \text{SS}(\Psi - \Gamma \mathbf{C}^* - \Gamma \mathbf{B}^* - \mathbf{S}) + \lambda_C \mathbf{1}' |\mathbf{C}^{*\tau}| \mathbf{1} + \lambda_B \mathbf{1}' |\mathbf{B}^{*\tau}| \mathbf{1}, \quad (11)$$

where $\text{SS}(\mathbf{T}) = \text{trace}(\mathbf{T}'\mathbf{T})$, $\mathbf{C}^* = [\mathbf{C}, \mathbf{0}]$ and $\mathbf{B}^* = [\mathbf{0}, \mathbf{B}]$ such that $\mathbf{A} = \mathbf{C}^* + \mathbf{B}^*$. The proposed algorithm begins by assigning initial values to the parameters and then repeats five steps at each iteration until convergence. In each step, each set of parameters— $\mathbf{C}$, $\mathbf{B}$, $\mathbf{W}$, $\mathbf{U}$, or $\mathbf{D}$ —is updated while the others are held fixed. Specifically, $\mathbf{C}$ and $\mathbf{B}$ are updated in the first and second steps, respectively, followed by updates to $\mathbf{W}$, $\mathbf{U}$, and $\mathbf{D}$ in the remaining steps. These latter steps remain identical to those in IGSCA (Hwang, Cho, Jung, et al., 2021; also see Hwang et al., 2017).

Step 1: Update $\mathbf{C}$ for fixed $\mathbf{B}$, $\mathbf{W}$, $\mathbf{U}$, and $\mathbf{D}$. This is equivalent to minimizing

$$\begin{aligned} \phi_1 &= \frac{\tau}{2} \text{SS}(\Psi^* - \Gamma \mathbf{C}^*) + \lambda_C \mathbf{1}' |\mathbf{C}^{*\tau}| \mathbf{1} \\ &= \frac{\tau}{2} \text{SS}(\text{vec}(\Psi^*) - (\mathbf{I}_{J+P} \otimes \Gamma) \text{vec}(\mathbf{C}^*)) + \lambda_C \mathbf{1}' |\text{vec}(\mathbf{C}^*)^\tau| \\ &= \frac{\tau}{2} \text{SS}(\mathbf{y} - \mathbf{X}\mathbf{c}) + \lambda_C \mathbf{1}' |\mathbf{c}^\tau| \end{aligned} \quad (12)$$

where $\mathbf{y} = \text{vec}(\Psi^*) = \text{vec}(\Psi - \Gamma \mathbf{B}^* - \mathbf{S})$, $\mathbf{c}$ is the vector consisting of any non-zero loadings in $\text{vec}(\mathbf{C}^*)$, and $\mathbf{X}$ is the matrix of the columns of $\mathbf{I}_{J+P} \otimes \Gamma$ corresponding to the non-zero loadings in $\text{vec}(\mathbf{C}^*)$. If $\tau = 1$, we can update $\mathbf{c}$ using the coordinate descent algorithm (Friedman et al., 2007), a computationally efficient and general optimization method applicable to a wide range of separable and (quasi-)convex penalization problems (e.g., Breheny & Huang, 2011; Wu & Lange, 2008). In this algorithm, each element of $\mathbf{c}$, denoted by c_k , is updated holding the other elements constant. Let $\mathbf{c}_{(-k)}$ denote $\mathbf{c}$, whose k th element is zero, and $\mathbf{x}_k$ denote the k th column of $\mathbf{X}$. The objective function (12) can be rewritten as

$$\begin{aligned}
\varphi_1 &= \frac{1}{2} \text{SS}([\mathbf{y} - \mathbf{X}\mathbf{c}_{(-k)}] - \mathbf{x}_k c_k) + \lambda_C \mathbf{1}' |\mathbf{c}_{(-k)}| + \lambda_C |c_k| \\
&= \frac{1}{2} \sum_{i=1}^n (f_i - x_{ik} c_k)^2 + \lambda_C \mathbf{1}' |\mathbf{c}_{(-k)}| + \lambda_C |c_k|
\end{aligned} \tag{13}$$

where f_i is the i th element of $\mathbf{y} - \mathbf{X}\mathbf{c}_{(-k)}$, x_{ik} is the i th element of $\mathbf{x}_k$, and $n = N(J+P)$. Minimizing (13) with respect to c_k is equivalent to minimizing

$$\varphi_1^* = \frac{1}{2} \sum_{i=1}^n (f_i - x_{ik} c_k)^2 + \lambda_C |c_k|. \tag{14}$$

Let $\bar{c}_k$ denote the ordinary least squares estimate of c_k , given by $\bar{c}_k = \sum_{i=1}^n x_{ik} f_i / \sum_{i=1}^n x_{ik}^2$. The lasso

estimate of c_k is then obtained as follows.

$$\hat{c}_k = \begin{cases} \bar{c}_k - \lambda_C / \sum_{i=1}^n x_{ik}^2, & \text{if } \bar{c}_k > 0 \text{ and } \lambda_C / \sum_{i=1}^n x_{ik}^2 < |\bar{c}_k| \\ \bar{c}_k + \lambda_C / \sum_{i=1}^n x_{ik}^2, & \text{if } \bar{c}_k < 0 \text{ and } \lambda_C / \sum_{i=1}^n x_{ik}^2 < |\bar{c}_k| \\ 0, & \text{if } \lambda_C / \sum_{i=1}^n x_{ik}^2 \geq |\bar{c}_k| \end{cases} \tag{15}$$

If $\tau = 2$, minimizing (12) with respect to $\mathbf{c}$ is equivalent to minimizing

$$\varphi_1 = \text{SS}(\mathbf{y} - \mathbf{X}\mathbf{c}) + \lambda_C \mathbf{c}'\mathbf{c}. \tag{16}$$

The ridge estimate of $\mathbf{c}$ is then obtained by

$$\hat{\mathbf{c}} = (\mathbf{X}'\mathbf{X} + \lambda_C \mathbf{I})^{-1} \mathbf{X}'\mathbf{y}. \tag{17}$$

The updated $\mathbf{C}$ is reconstructed from $\hat{\mathbf{c}}$.

Step 2: Update $\mathbf{B}$ for fixed $\mathbf{C}$, $\mathbf{W}$, $\mathbf{U}$, and $\mathbf{D}$. This is equivalent to minimizing

$$\begin{aligned}
\phi_2 &= \frac{\tau}{2} \text{SS}(\mathbf{\Psi}^* - \mathbf{\Gamma} \mathbf{B}^*) + \lambda_B \mathbf{1}' |\mathbf{B}^*| \mathbf{1} \\
&= \frac{\tau}{2} \text{SS}(\text{vec}(\mathbf{\Psi}^*) - (\mathbf{I}_{J+P} \otimes \mathbf{\Gamma}) \text{vec}(\mathbf{B}^*)) + \lambda_B \mathbf{1}' |\text{vec}(\mathbf{B}^*)| \mathbf{1} \\
&= \frac{\tau}{2} \text{SS}(\mathbf{y} - \mathbf{X} \mathbf{b}) + \lambda_B \mathbf{1}' |\mathbf{b}| \mathbf{1}
\end{aligned} \tag{18}$$

where $\mathbf{y} = \text{vec}(\mathbf{\Psi}^*) = \text{vec}(\mathbf{\Psi} - \mathbf{\Gamma} \mathbf{C}^* - \mathbf{S})$, $\mathbf{b}$ is the vector of any non-zero path coefficients in $\text{vec}(\mathbf{B}^*)$, and $\mathbf{X}$ now denotes the matrix of the columns of $\mathbf{I}_{J+P} \otimes \mathbf{\Gamma}$ corresponding to the non-zero path coefficients in $\text{vec}(\mathbf{B}^*)$. Then the lasso or ridge estimate of $\mathbf{b}$, denoted by $\hat{\mathbf{b}}$, can be obtained in a manner similar to (15) or (17), respectively, replacing λ_C with λ_B . The updated $\mathbf{B}$ is reconstructed from $\hat{\mathbf{b}}$.

Step 3: Update $\mathbf{W}$ (or equivalently $\mathbf{\Gamma}$) for fixed $\mathbf{C}$, $\mathbf{B}$, $\mathbf{U}$, and $\mathbf{D}$. Since $\mathbf{W}$ is not regularized, this step is equivalent to minimizing the same objective function (9) as in IGSCA. As discussed in Hwang et al. (2021), $\mathbf{W}$ can be updated by $\hat{\mathbf{W}} = \mathbf{M} \mathbf{H}' (\mathbf{H} \mathbf{H}')^{-1}$, where $\mathbf{H} = \mathbf{A} - [\mathbf{0}, \mathbf{I}_P]$, and $\mathbf{M} = [\mathbf{I}_J, \mathbf{0}]$. Each column of $\mathbf{\Gamma}$ is subsequently updated by $\hat{\boldsymbol{\gamma}}_p = \mathbf{Z}_p^* \mathbf{w}_p$, where $\mathbf{Z}_p^*$ includes the columns of $\mathbf{Z} - \mathbf{U} \mathbf{D}$ corresponding to a set of indicators associated with the p th column, and $\mathbf{w}_p$ contains the elements of the p th column of $\mathbf{W}$ assigned to $\mathbf{Z}_p^*$. Subsequently, it is standardized to

satisfy $\hat{\boldsymbol{\gamma}}_p' \hat{\boldsymbol{\gamma}}_p = N$, which can be done by multiplying $\mathbf{w}_p$ by $\sqrt{\frac{N}{\mathbf{w}_p' \mathbf{Z}_p^* \mathbf{Z}_p^* \mathbf{w}_p}}$.

Step 4: Update $\mathbf{U}$ for fixed $\mathbf{C}$, $\mathbf{B}$, $\mathbf{W}$, and $\mathbf{D}$. Since $\mathbf{U}$ is not subject to regularization, this step is again equivalent to minimizing the objective function (9). Following Hwang et al. (2021), $\mathbf{U}$ can

be updated as follows. Let $\mathbf{\Gamma} = \mathbf{Q} \mathbf{R} = \begin{bmatrix} \tilde{\mathbf{\Gamma}} & \mathbf{\Gamma}_\perp \end{bmatrix} \begin{bmatrix} \mathbf{I} \\ \mathbf{O} \end{bmatrix}$ denote the QR decomposition of $\mathbf{\Gamma}$, where $\mathbf{\Gamma}_\perp$

is an N by $N-P$ orthonormal basis matrix of the null space of Γ . Let $\text{SVD}(\Gamma_{\perp}' \mathbf{Z} \mathbf{D}) = \mathbf{P} \mathbf{\Lambda} \mathbf{T}'$.

Then, $\mathbf{U}$ is updated by $\hat{\mathbf{U}} = \Gamma_{\perp} \mathbf{P} \mathbf{T}'$.

Step 5: Update $\mathbf{D}$ for fixed $\mathbf{C}$, $\mathbf{B}$, $\mathbf{W}$, and $\mathbf{U}$. Again, since $\mathbf{D}$ is not regularized, this step involves minimizing the same objective function (9). As described in Hwang et al. (2021), $\mathbf{D}$ is updated by $\hat{\mathbf{D}} = \text{diag}(\mathbf{U}' \mathbf{Z})$. Any elements of $\mathbf{D}$ corresponding to components are then set to zero, resulting in the updated $\mathbf{S}$.

In Steps 1 and 2, it is worth noting that it is possible to regularize only part of the loadings and path coefficients. For example, in (12), we can divide the non-zero loading vector $\mathbf{c}$ into two sub-vectors, i.e., $\mathbf{c} = [\mathbf{c}_1; \mathbf{c}_2]$, where $\mathbf{c}_1$ consists of loadings to be regularized only and $\mathbf{c}_2$ includes the remaining loadings that remain unregularized. Then, $\mathbf{X}$ can be rearranged as $[\mathbf{X}_1, \mathbf{X}_2]$, where $\mathbf{X}_1$ includes the columns of $\mathbf{X}$ corresponding to $\mathbf{c}_1$ and $\mathbf{X}_2$ includes the columns of $\mathbf{X}$ corresponding to $\mathbf{c}_2$. Then, we can re-express (12) as

$$\begin{aligned} \phi_1 &= \frac{\tau}{2} \text{SS}(\mathbf{y} - [\mathbf{X}_1, \mathbf{X}_2] \begin{bmatrix} \mathbf{c}_1 \\ \mathbf{c}_2 \end{bmatrix}) + \lambda_C \mathbf{1}' |\mathbf{c}_1^{\tau}| \\ &= \frac{\tau}{2} \text{SS}(\mathbf{y} - \mathbf{X}_1 \mathbf{c}_1 - \mathbf{X}_2 \mathbf{c}_2) + \lambda_C \mathbf{1}' |\mathbf{c}_1^{\tau}|. \end{aligned} \quad (19)$$

Thus, $\mathbf{c}_2$ can be updated via ordinary least squares, whereas $\mathbf{c}_1$ can be updated as discussed in Step 1. The same procedure can be applied to regularize only part of the path coefficients.

We can apply K -fold cross-validation (e.g., Hastie et al., 2009) to determine the values of λ_C and λ_B . In K -fold cross-validation, the entire dataset is divided into K subsets. One of the K subsets is used as a validation sample, while the remaining $K-1$ subsets are used as a training sample to estimate parameters. Given the values of λ_C and λ_B , parameter estimates are obtained from the training sample and are subsequently applied to the validation sample to calculate the prediction error of the specified model. This procedure is repeated K times, changing the

validation and training samples. Then, the cross-validation estimate of prediction error is computed over all K validation samples.

Specifically, let $\mathbf{Z}^{(k)}$ denote the k -th validation sample of $\mathbf{Z}$ ($k = 1, \dots, K$) and let $\mathbf{Z}^{(-k)}$ denote the corresponding training sample obtained by removing $\mathbf{Z}^{(k)}$ from $\mathbf{Z}$. Regularized IGSCA is then applied to $\mathbf{Z}^{(-k)}$ to estimate the model parameters. Let $\mathbf{W}^{(-k)}$, $\mathbf{A}^{(-k)}$, and $\mathbf{D}^{(-k)}$ denote the estimated $\mathbf{W}$, $\mathbf{A}$, and $\mathbf{D}$ matrices based on $\mathbf{Z}^{(-k)}$. Let $\mathbf{U}^{(k)}$ denote the matrix of unique variables for $\mathbf{Z}^{(k)}$, obtained using $\mathbf{W}^{(-k)}$ and $\mathbf{D}^{(-k)}$, as described in Step 4. Then, the cross-validation estimate of prediction error (PE) under fixed values of λ_C and λ_B is obtained by

$$\text{PE}(\lambda_C, \lambda_B) = \frac{1}{K} \sum_{k=1}^K \text{SS}(\Psi^{(k)} - \Gamma^{(k)} \mathbf{A}^{(-k)} - \mathbf{S}^{(k)}), \quad (20)$$

where $\Psi^{(k)} = [\mathbf{Z}^{(k)}, \mathbf{Z}^{(k)} \mathbf{W}^{(-k)}]$, $\Gamma^{(k)} = \mathbf{Z}^{(k)} \mathbf{W}^{(-k)}$, and $\mathbf{S}^{(k)} = \mathbf{U}^{(k)} \mathbf{D}^{(-k)}$. The values of (20) are repeatedly calculated with the values of λ_C and λ_B systematically varied. The values of λ_C and λ_B associated with the smallest value of (20) are selected as the final ones. In practice, $K = 5$ or 10 is typically recommended (Hastie et al., 2009, p. 243), although $K = N$ (i.e., leave-one-out cross-validation) can be useful in small samples.

3. Simulation Study

We conducted a Monte Carlo simulation study to evaluate the performance of regularized IGSCA. In this study, we focused on how well regularized IGSCA could recover parameters. Figure 2 displays the population model specified for the study, where twelve exogenous components, signified by hexagons, influenced an endogenous factor, denoted by a circle. Each component or factor was associated with three indicators. This model was to encompass the one considered in the empirical application section. We fixed the loadings per factor/component as 0.6, 0.7, and 0.8. We assigned non-zero values to seven path coefficients, each with varying

effect sizes and signs: $b_1 = 0.5$, $b_3 = -0.3$, $b_5 = 0.1$, $b_7 = -0.05$, $b_9 = -0.5$, $b_{11} = 0.3$, $b_{13} = -0.1$, and $b_{15} = 0.05$. We set the rest of the path coefficients to zero: $b_2 = b_4 = b_6 = b_8 = b_{10} = b_{12} = b_{14} = 0$.

===== Insert Figure 2 around here =====

We manipulated three experimental factors: sample size ($N = 100, 200, 400, 800$, and 1600), the correlations among the exogenous components ($r = 0.1, 0.3, 0.5$, and 0.7), and the normality of indicators (normal vs. non-normal). The first two experimental factors were known to influence the performance of IGSCA (Hwang, Cho, Jung, et al., 2021). The third factor was included to examine the behavior of regularized IGSCA under non-normal distributions. Specifically, we employed a skewed normal distribution with a skewness of 1.25 , as used in the simulation study by Hwang et al. (2010). Other experimental factors frequently considered in SEM simulations—such as model complexity and model misspecification (Bandalos & Gagné, 2012; Paxton et al., 2001)—were not included, as they have been shown to have limited impact on IGSCA's performance (Hwang, Cho, Jung, et al., 2021). We fully crossed the three experimental factors, resulting in 40 conditions. For each condition, we generated 500 random samples from either a multivariate normal or skewed-normal distribution with zero means and the covariance matrix implied by the parameter values, according to the data generation procedure described in Hwang et al. (2021; see also Cho & Choi, 2019).

We analyzed the simulated data using (non-regularized) IGSCA and regularized IGSCA. For regularized IGSCA, we applied L_1 and L_2 regularization separately to the path coefficients only since the measurement model included a small number of indicators per factor/component with large loadings. This made multicollinearity and indicator selection less of a concern in the measurement model. We used 5-fold cross-validation to select the optimal λ_B value from a grid of candidate values. For the Lasso penalty, we used values ranging from 0 (no regularization) to

1, increasing on a logarithmic scale (e.g., 0, 0.01, 0.0126, 0.0158, ..., 1). For Ridge, we used a similarly sized grid ranging from 0 to 10, also spaced logarithmically (e.g., 0, 0.1, 0.126, 0.158, ..., 10). These grids allow for a fine-grained search over the strength of regularization. We developed MATLAB code for IGSCA and its regularized variants and employed it—together with the *TestSEM* package (Cho, 2025)—to perform the simulation study¹.

Once the simulation was complete, we calculated the empirical biases and root mean squared errors (RMSEs) of the three estimators for each individual parameter within each condition. We then took the absolute values of these biases and averaged them across loadings, nonzero path coefficients, and zero path coefficients separately for each condition. These results for all individual conditions are provided in the Supplementary Material. Since there were no substantial differences between the results under normal and non-normal conditions, we focus here on reporting the averages across distributional conditions.

Table 1 presents the average biases of the loading estimators obtained from IGSCA, L1-regularized IGSCA, and L2-regularized IGSCA under each condition. Across all conditions, IGSCA and both regularized variants yielded virtually identical average biases, all of which were close to zero. This result is expected to some extent: although loading estimation in the IGSCA framework is not fully independent of the structural model, no regularization was imposed on the loadings in this study (i.e., $\lambda_C = 0$), meaning that both IGSCA and regularized IGSCA estimated the loadings in the same manner. As a result, these methods also produced nearly identical root mean squared errors (RMSEs) for the loading estimators. This finding aligns with the results of Hwang et al. (2021), who demonstrated the unbiasedness of IGSCA estimators for both component and factor loadings.

¹ The code is publicly available at: https://osf.io/gb7ra/?view_only=f3976c9315dc494ab43d0c0121c68043.

===== Insert Table 1 =====

Table 2 presents the average bias and RMSE values of the IGSCA and L_1 - and L_2 -regularized IGSCA estimators for non-zero path coefficients. Overall, the IGSCA estimators were unbiased across all conditions, aligning with the findings of Hwang et al. (2021). In contrast, both regularized versions exhibited larger biases than IGSCA, though these differences diminished as sample size increased. Specifically, the L_1 - and L_2 -regularized IGSCA estimators showed larger biases when the correlations among exogenous components were higher. This pattern reflects the well-known behavior of regularization methods, where stronger correlations among predictors lead to more aggressive shrinkage, thereby introducing greater bias in exchange for substantial variance reduction (e.g., Xu et al., 2012; Zhao & Yu, 2006).

In large samples (e.g., $N \geq 800$), however, the bias of the L_2 -regularized IGSCA estimators largely disappeared, and its performance became nearly indistinguishable from that of IGSCA. This finding aligns with the expectation that the influence of the L_2 regularization diminishes with increasing sample size, particularly when the tuning parameter is selected via cross-validation (e.g., Takane & Hwang, 2007). Despite being biased on average, both L_1 - and L_2 -regularized IGSCA tended to yield smaller RMSEs than IGSCA across conditions—especially under small sample sizes and/or higher correlations among exogenous components.

===== Insert Table 2 around here =====

Table 3 presents the average bias and RMSE values of the IGSCA and L_1 - and L_2 -regularized IGSCA estimators for zero path coefficients. In contrast to the previous case, all estimators were virtually unbiased across all conditions. This result is expected because when the true parameter is zero, shrinkage induced by regularization aligns with the true value, producing little to no bias. Consequently, the differences in RMSE between IGSCA and its regularized

counterparts became more pronounced. The L_1 - and L_2 -regularized IGSCA estimators consistently achieved smaller RMSE values than IGSCA, particularly in conditions with smaller samples and/or stronger correlations among exogenous components.

===== Insert Table 3 around here =====

As an anonymous reviewer suggested, we additionally investigated the performance of the L_1 -regularized IGSCA estimator in identifying non-zero path coefficients. For each sample, we treated any estimated path coefficient fixed to zero as a rejection. For each experimental condition, we calculated the rejection rate for each path coefficient and then averaged them within groups of path coefficients sharing the same absolute size (i.e., .5, .3, .1, .05, and 0). Since there were no substantial differences between the results under normal and non-normal distributions, we report the average rejection rates across distributional conditions. Detailed results for all individual conditions are available in the Supplementary Material.

Table 4 presents the average rejection rates of the L_1 -regularized IGSCA estimators across five groups of path coefficients. Overall, L_1 -regularized IGSCA frequently failed to retain non-zero path coefficients, particularly when the sample size was small ($N \leq 200$), the correlations among exogenous components were high, and/or the true path coefficients were small. However, the rejection rates for nonzero path coefficients decreased as the sample size increased. In large sample ($N = 1600$), the rejection rates for path coefficients of moderately small to large size (e.g., ≥ 0.1) approached zero across all conditions. In contrast, path coefficients of very small magnitude (i.e., 0.05) continued to show non-negligible rejection rates, particularly when the correlations among exogenous components were high (e.g., rejection rate = 0.31 for $r = 0.7$).

For zero-path coefficients, L_1 -regularized IGSCA was not consistently effective at rejecting the zero path coefficients. Across all conditions, the rejection rates for zero path coefficients remained between .4 and .6 and did not improve with increasing sample size. This may be because even though the estimated coefficients for zero paths were shrunk toward zero (see Table 3 for near-zero bias), they were not sufficiently penalized to be set exactly to zero. These findings are consistent with prior literature showing that L_1 -regularized estimators often struggle to exclude true-zero coefficients in regression models (e.g., Devlin et al., 2003; He & Lin, 2011). This highlights the need for an additional screening or thresholding step if L_1 -regularized IGSCA is to be used as a specification search algorithm.

In sum, both L_1 - and L_2 -regularized IGSCA produced more biased yet more efficient estimates than IGSCA. As a result, the regularized estimators were closer, on average, to the true parameter values across most conditions. On the other hand, while L_1 -regularized IGSCA showed some promise in identifying nonzero path coefficients, it should be used with caution as a specification search procedure due to its limited ability to fully exclude zero path coefficients.

4. Empirical Application

This example is part of behavioral and neuroimaging data collected from 1200 healthy adults aged 22-35, provided by the Human Connectome Project (HCP) project (<https://www.humanconnectome.org/study/hcp-young-adult>). We examined how the volumes of nine ROIs in the subcortical areas of the brain influence psychological well-being. Subcortical areas are known to play important roles in cognitive, affective, and social functions in humans (Koshiyama et al., 2018). We represented each ROI as a component or a weighted composite of its left and right portions (i.e., two indicators per ROI) and psychological well-being as a factor

underlying three indicators, as shown in the measurement model in Table 3. We also included age, gender (0 = M, 1 = F), and total intracranial volume (TIV) as covariates for psychological well-being. Figure 3 displays the hypothesized structural model. The sample size was 1112 after removing all cases with missing data.

===== Insert Figure 3 around here =====

We applied (non-regularized) IGSCA and L_1 - and L_2 -regularized IGSCA to fit the model to the data. We employed five-fold cross-validation to determine the values of λ_C and λ_B for both types of regularization, varying each of their values from 0.001 to 1, generated on a logarithmic scale in increments of 0.04. For both L_1 - and L_2 -regularized IGSCA, $\lambda_C = 0$, indicating no regularization on the loadings. This could be expected because the measurement model contained a small number of indicators per component/factor and was well-specified. Conversely, $\lambda_B = 0.0275$ for L_1 -regularized IGSCA and $\lambda_B = 0.4786$ for L_2 -regularized IGSCA. We used the bootstrap method (Efron, 1979) with 4000 bootstrap samples to calculate the standard errors for IGSCA and L_2 -regularized IGSCA. Standard errors for the L_1 -regularized IGSCA are not reported, as they cannot be validly estimated using the standard bootstrap method (e.g., Chatterjee & Lahiri, 2011).

Table 5 provides the loading estimates and their standard errors (SEs) obtained from the three methods. All loading estimates were large with small standard errors. Both types of regularized IGSCA provided virtually identical loading estimates to those from IGSCA, as the loadings were not regularized.

Table 6 exhibits the path coefficient estimates and their standard errors obtained from the methods. The estimates from IGSCA were all non-zero, with many being close to zero. Notably, the volume of the ROI caudate had a relatively large and negative impact on psychological well-

being ($b = -0.15$, $SE = 0.06$). This is consistent with prior research indicating that well-being is negatively associated with the caudate's volume (e.g., Hermes et al., 2011; Jamshidi et al., 2022). The covariates gender and total intracranial volume also had a relatively large effect on psychological well-being ($b = 0.13$, $SE = 0.05$ for gender; $b = 0.19$, $SE = 0.06$ for total intracranial volume).

As expected, L_1 - and L_2 -regularized IGSCA tended to shrink IGSCA's estimates toward zero. Specifically, relatively small estimates, including those for lateral ventricle and putamen, were reduced to zero in L_1 -regularized IGSCA or near zero in L_2 -regularized IGSCA. Relatively large estimates, such as those for the caudate and total intracranial volume, also become smaller, though they remained far from zero. The standard errors of the path coefficient estimates are substantially smaller in L_1 - and L_2 -regularized IGSCA compared to IGSCA, suggesting that the regularized estimates are likely to be more efficient than non-regularized ones. The interpretation of the regularized estimates remained the same as the non-regularized counterparts. It is noteworthy that L_1 -regularized IGSCA led the path coefficient estimates for all ROIs to zero, except for the caudate. This resulted in a more parsimonious structural model that is easier to interpret and aligns with previous findings (e.g., Hermes et al., 2011; Jamshidi et al., 2022).

5. Concluding Remarks

We developed a method that combined IGSCA with L_1 and L_2 regularization in a unified framework. This regularized IGSCA can handle multicollinearity problems by the adoption of L_1 regularization as well as address variable selection through L_2 regularization.

In our simulation study, we found that both L_1 - and L_2 -regularized IGSCA generally resulted in more accurate parameters than IGSCA. Specifically, while regularization introduced

modest bias in the estimation of nonzero path coefficients—particularly under small samples and high multicollinearity—it consistently reduced RMSE compared to the unregularized IGSCA estimator. This efficiency gain was especially notable when true effects were small or when predictors were highly correlated, reflecting the expected bias–variance trade-off of regularization. Moreover, both L_1 - and L_2 -regularized IGSCA produced virtually unbiased estimates for loadings and zero path coefficients across all conditions, like IGSCA.

On the other hand, L_1 -regularized IGSCA effectively identified non-zero coefficients, especially when sample sizes are moderate to large. However, it also showed limited ability to shrink zero coefficients fully to zero, even in large samples, underscoring the need for additional model selection steps when using it for specification search (e.g., Laurin et al., 2016). Our real data analysis further demonstrated the utility of L_1 -regularized IGSCA in simplifying the model by automatically selecting a brain region relevant to psychological well-being—a finding supported by prior research. Nonetheless, the selected regions with small effect sizes should be interpreted with caution.

Taken together, these results suggest that regularized IGSCA offer a more efficient approach to parameter recovery in high-dimensional or small-sample contexts, though careful interpretation is warranted when using it for structural model selection. In practice, regularized IGSCA may be used in lieu of IGSCA. If multicollinearity is of concern, regularized GSCA with L_2 regularization can be used to yield more accurate estimates than IGSCA. In the absence of multicollinearity, regularized IGSCA will reduce to IGSCA, setting both tuning parameters to zero. If variable selection is of main interest, regularized IGSCA with L_1 regularization can be adopted to select subsets of indicators or components/factors, making the model simpler and more interpretable. When the model includes only relevant variables, regularized IGSCA will

produce identical results to those of IGSCA, again selecting zero tuning parameters. However, it is important to note that regularized IGSCA is computationally more costly than IGSCA because it involves cross-validation for the selection of the tuning parameters.

Future research may extend regularized IGSCA to further enhance its flexibility. For example, researchers may combine the method with the group lasso (Yuan & Lin, 2006), which allows (often pre-defined) groups of independent variables to be selected in linear regression. Regularized IGSCA with L_1 regularization is currently to select variables individually. However, it is sometimes more sensible to choose a group of variables as a whole. For example, in the analysis of variance settings, a group of dummy variables represents an independent variable. In this case, it would be desirable to select a group of dummy variables rather than individual dummy variables in that group. The group lasso may be more useful for selecting important and pre-defined groups of dummy variables, each of which represents an independent variable, than the lasso. Similarly, a group of indicators is typically associated with a component or factor in SEM. Thus, it may be more effective to apply the group lasso if we are interested in selecting a group of relevant indicators rather than individual indicators per component or factor.

In addition, we may consider extending regularized IGSCA to the analysis of functions, whose responses are sequenced along a continuum such as time, frequency, or spatial location (e.g., Ramsay & Silverman, 2005). For example, when observed responses are assumed to come from a smooth data function of time, loadings in the measurement model should also be specified as a function of time, indicating the influence of a component or factor on the data function varying over time. Several lasso variants have been developed for functional linear regression (e.g., Gertheiss et al., 2013; Pannu & Billor, 2017; Zhao et al., 2015). It would be interesting to

investigate their feasibility of discarding the sub-intervals of a data function, which exhibit negligible relations with a component or factor in the measurement model.

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Table 1. The average absolute bias and RMSE values of the IGSCA and L_1 - and L_2 -regularized IGSCA estimators for loading parameters in different simulation conditions. Basic = IGSCA, Lasso = L_1 -regularized IGSCA, and Ridge = L_2 -regularized IGSCA.

		Loadings					
		Absolute Bias			RMSE		
r	N	Basic	Lasso	Ridge	Basic	Lasso	Ridge
0.1	100	0.00	0.00	0.00	0.04	0.04	0.04
	200	0.00	0.00	0.00	0.03	0.03	0.03
	400	0.00	0.00	0.00	0.02	0.02	0.02
	800	0.00	0.00	0.00	0.02	0.02	0.02
	1600	0.00	0.00	0.00	0.01	0.01	0.01
0.3	100	0.00	0.00	0.00	0.04	0.04	0.04
	200	0.00	0.00	0.00	0.03	0.03	0.03
	400	0.00	0.00	0.00	0.02	0.02	0.02
	800	0.00	0.00	0.00	0.02	0.02	0.02
	1600	0.00	0.00	0.00	0.01	0.01	0.01
0.5	100	0.00	0.00	0.00	0.04	0.04	0.04
	200	0.00	0.00	0.00	0.03	0.03	0.03
	400	0.00	0.00	0.00	0.02	0.02	0.02
	800	0.00	0.00	0.00	0.02	0.02	0.02
	1600	0.00	0.00	0.00	0.01	0.01	0.01
0.7	100	0.00	0.00	0.00	0.04	0.04	0.04
	200	0.00	0.00	0.00	0.03	0.03	0.03
	400	0.00	0.00	0.00	0.02	0.02	0.02
	800	0.00	0.00	0.00	0.02	0.02	0.02
	1600	0.00	0.00	0.00	0.01	0.01	0.01

Table 2. The average absolute bias and RMSE values of the IGSCA and L_1 - and L_2 -regularized IGSCA estimators for non-zero path coefficients in different simulation conditions. Basic = IGSCA, Lasso = L_1 -regularized IGSCA, and Ridge = L_2 -regularized IGSCA.

		Nonzero Path Coefficients					
		Absolute Bias			RMSE		
r	N	Basic	Lasso	Ridge	Basic	Lasso	Ridge
0.1	100	0.00	0.05	0.04	0.08	0.08	0.08
	200	0.00	0.03	0.02	0.05	0.06	0.06
	400	0.00	0.02	0.00	0.04	0.04	0.04
	800	0.00	0.01	0.00	0.03	0.03	0.03
	1600	0.00	0.01	0.00	0.02	0.02	0.02
0.3	100	0.00	0.06	0.05	0.10	0.10	0.10
	200	0.00	0.04	0.03	0.07	0.07	0.07
	400	0.00	0.03	0.01	0.05	0.05	0.05
	800	0.00	0.02	0.00	0.03	0.04	0.03
	1600	0.00	0.01	0.00	0.02	0.03	0.02
0.5	100	0.00	0.08	0.08	0.13	0.13	0.13
	200	0.00	0.05	0.05	0.09	0.09	0.09
	400	0.00	0.04	0.03	0.06	0.07	0.07
	800	0.00	0.03	0.00	0.04	0.05	0.04
	1600	0.00	0.02	0.00	0.03	0.04	0.03
0.7	100	0.01	0.11	0.11	0.19	0.18	0.17
	200	0.01	0.07	0.07	0.13	0.13	0.13
	400	0.00	0.05	0.05	0.09	0.09	0.10
	800	0.00	0.04	0.01	0.06	0.07	0.07
	1600	0.00	0.03	0.00	0.04	0.05	0.04

Table 3. The average absolute bias and RMSE values of the IGSCA and L_1 - and L_2 -regularized IGSCA estimators for zero path coefficients in different simulation conditions. Basic = IGSCA, Lasso = L_1 -regularized IGSCA, and Ridge = L_2 -regularized IGSCA.

		Zero Path Coefficients					
		Absolute Bias			RMSE		
r	N	Basic	Lasso	Ridge	Basic	Lasso	Ridge
0.1	100	0.00	0.00	0.00	0.08	0.05	0.07
	200	0.00	0.00	0.00	0.06	0.04	0.05
	400	0.00	0.00	0.00	0.04	0.03	0.04
	800	0.00	0.00	0.00	0.03	0.02	0.03
	1600	0.00	0.00	0.00	0.02	0.01	0.02
0.3	100	0.00	0.00	0.00	0.10	0.06	0.08
	200	0.00	0.00	0.00	0.07	0.04	0.06
	400	0.00	0.00	0.00	0.05	0.03	0.05
	800	0.00	0.00	0.00	0.03	0.02	0.03
	1600	0.00	0.00	0.00	0.02	0.02	0.02
0.5	100	0.00	0.00	0.00	0.13	0.07	0.09
	200	0.00	0.00	0.00	0.09	0.05	0.07
	400	0.00	0.00	0.00	0.06	0.04	0.06
	800	0.00	0.00	0.00	0.04	0.03	0.04
	1600	0.00	0.00	0.00	0.03	0.02	0.03
0.7	100	0.01	0.00	0.00	0.19	0.09	0.10
	200	0.01	0.00	0.00	0.13	0.07	0.09
	400	0.00	0.00	0.00	0.09	0.05	0.07
	800	0.00	0.00	0.00	0.06	0.04	0.06
	1600	0.00	0.00	0.00	0.04	0.03	0.04

Table 4. The average rejection rates of the L_1 -regularized IGSCA estimators across five groups of path coefficients in different conditions.

r	N	$b = 0.5$	$b = 0.3$	$b = 0.1$	$b = 0.05$	$b = 0$
0.1	100	0.00	0.00	0.30	0.44	0.50
	200	0.00	0.00	0.12	0.33	0.45
	400	0.00	0.00	0.03	0.20	0.41
	800	0.00	0.00	0.00	0.10	0.39
	1600	0.00	0.00	0.00	0.02	0.40
0.3	100	0.00	0.03	0.37	0.49	0.52
	200	0.00	0.00	0.22	0.40	0.48
	400	0.00	0.00	0.09	0.31	0.45
	800	0.00	0.00	0.01	0.19	0.41
	1600	0.00	0.00	0.00	0.08	0.42
0.5	100	0.01	0.11	0.49	0.54	0.58
	200	0.00	0.01	0.33	0.47	0.52
	400	0.00	0.00	0.18	0.40	0.50
	800	0.00	0.00	0.06	0.29	0.46
	1600	0.00	0.00	0.00	0.17	0.45
0.7	100	0.17	0.37	0.63	0.67	0.68
	200	0.01	0.10	0.46	0.55	0.57
	400	0.00	0.01	0.32	0.47	0.53
	800	0.00	0.00	0.17	0.41	0.50
	1600	0.00	0.00	0.06	0.31	0.45

Table 5. The loading estimates and their standard errors from IGSCA and L_1 - and L_2 -regularized IGSCA in the empirical application. Basic = IGSCA, Lasso = L_1 -regularized IGSCA, and Ridge = L_2 -regularized IGSCA.

Indicator	Component/Factor	Basic		Lasso	Ridge	
		Estimate	SE	Estimate	Estimate	SE
Left-Lateral-Ventricle	Lateral ventricle	0.96	0.00	0.96	0.96	0.00
Right-Lateral-Ventricle		0.96	0.00	0.96	0.96	0.00
Left-Thalamus-Proper	Thalamus	0.96	0.00	0.96	0.96	0.00
Right-Thalamus-Proper		0.96	0.00	0.96	0.96	0.00
Left-Caudate	Caudate	0.99	0.00	0.99	0.99	0.00
Right-Caudate		0.98	0.00	0.98	0.98	0.00
Left-Putamen	Putamen	0.96	0.00	0.96	0.96	0.00
Right-Putamen		0.96	0.00	0.96	0.96	0.00
Left-Pallidum	Pallidum	0.93	0.01	0.93	0.93	0.00
Right-Pallidum		0.92	0.00	0.93	0.92	0.00
Left-Hippocampus	Hippocampus	0.96	0.01	0.96	0.96	0.01
Right-Hippocampus		0.96	0.01	0.96	0.96	0.01
Left-Amygdala	Amygdala	0.95	0.00	0.95	0.95	0.00
Right-Amygdala		0.95	0.00	0.95	0.95	0.00
Left-Accumbens-area	Nucleus	0.92	0.01	0.92	0.92	0.00
Right-Accumbens-area	Accumbens	0.92	0.01	0.92	0.92	0.00
Left-VentralDC	Ventral diencephalon	0.98	0.00	0.98	0.98	0.00
Right-VentralDC		0.98	0.00	0.98	0.98	0.00
Total intracranial volume	TIV	1.00	0.00	1.00	1.00	0.00
Gender	Gender	1.00	0.00	1.00	1.00	0.00
Age	Age	1.00	0.00	1.00	1.00	0.00
General Life Satisfaction	Psychological wellbeing	0.80	0.02	0.80	0.80	0.02
Meaning and Purpose		0.73	0.03	0.74	0.74	0.02
Positive Affect Survey		0.70	0.03	0.70	0.70	0.02

Table 6. The path coefficient estimates and their standard errors from IGSCA and L_1 - and L_2 -regularized IGSCA in the empirical application. Basic = IGSCA, Lasso = L_1 -regularized IGSCA, and Ridge = L_2 -regularized IGSCA.

Predictors	Basic		Lasso	Ridge	
	Estimate	SE	Estimate	Estimate	SE
Lateral ventricle	-0.01	0.04	0	0	0.02
Thalamus	-0.02	0.06	0	0	0.02
Caudate	-0.15	0.05	-0.08	-0.07	0.02
Putamen	0.04	0.06	0	0	0.02
Pallidum	-0.03	0.06	0	-0.01	0.02
Hippocampus	-0.02	0.06	0	0	0.02
Amygdala	0.05	0.06	0	0.02	0.02
Nucleus accumbens	-0.05	0.05	0	-0.03	0.02
Ventral diencephalon	0.02	0.06	0	0.01	0.02
TIV	0.19	0.06	0.08	0.06	0.02
Gender	0.13	0.05	0.05	0.06	0.02
Age	-0.03	0.03	0	-0.02	0.02

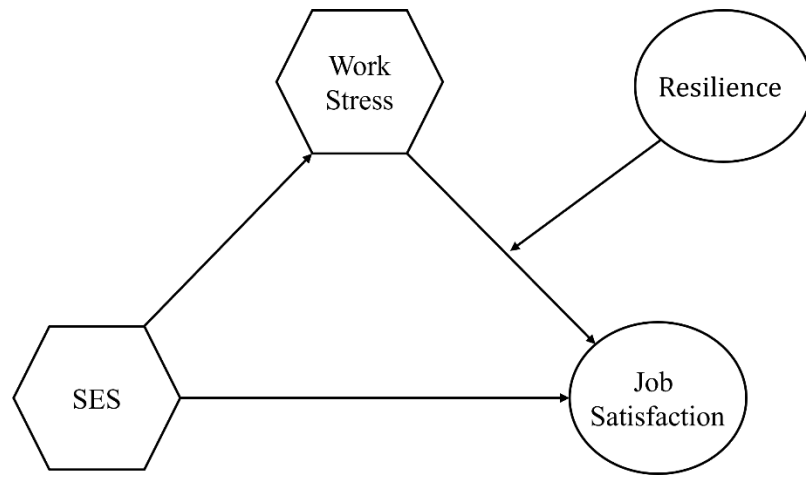


Figure 1. An exemplary structural model with both factors and components. Circles represent factors and hexagons represent components. All error terms are omitted.

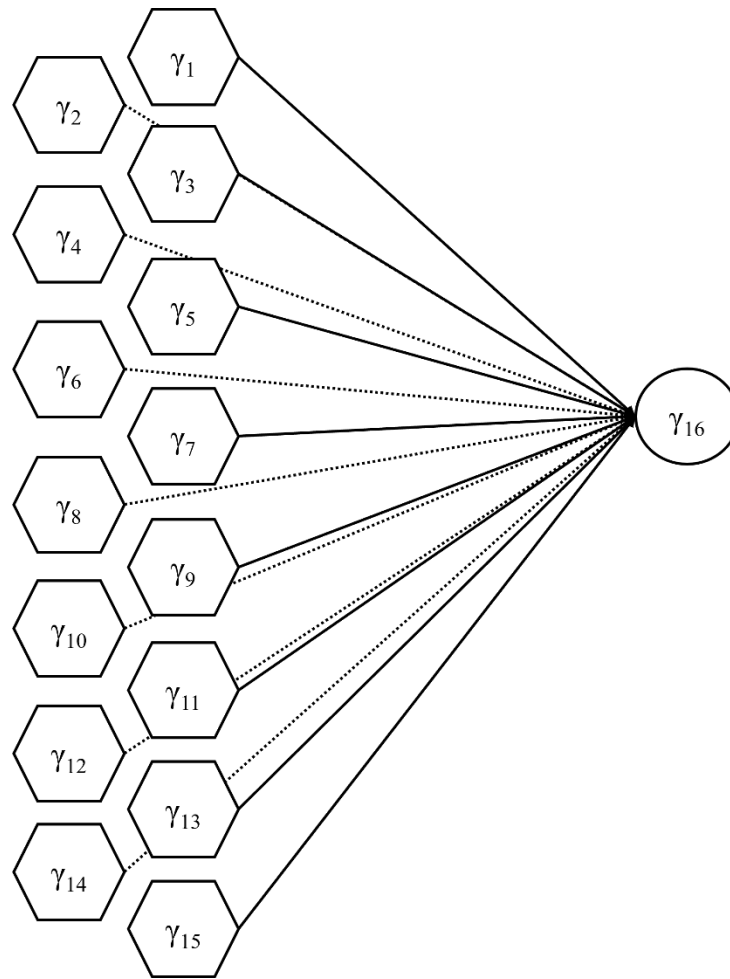


Figure 2. The structural model specified for the simulation study. A circle represents a factor, and hexagons represent components. Dashed arrows indicate zero path coefficients. The error term is omitted.

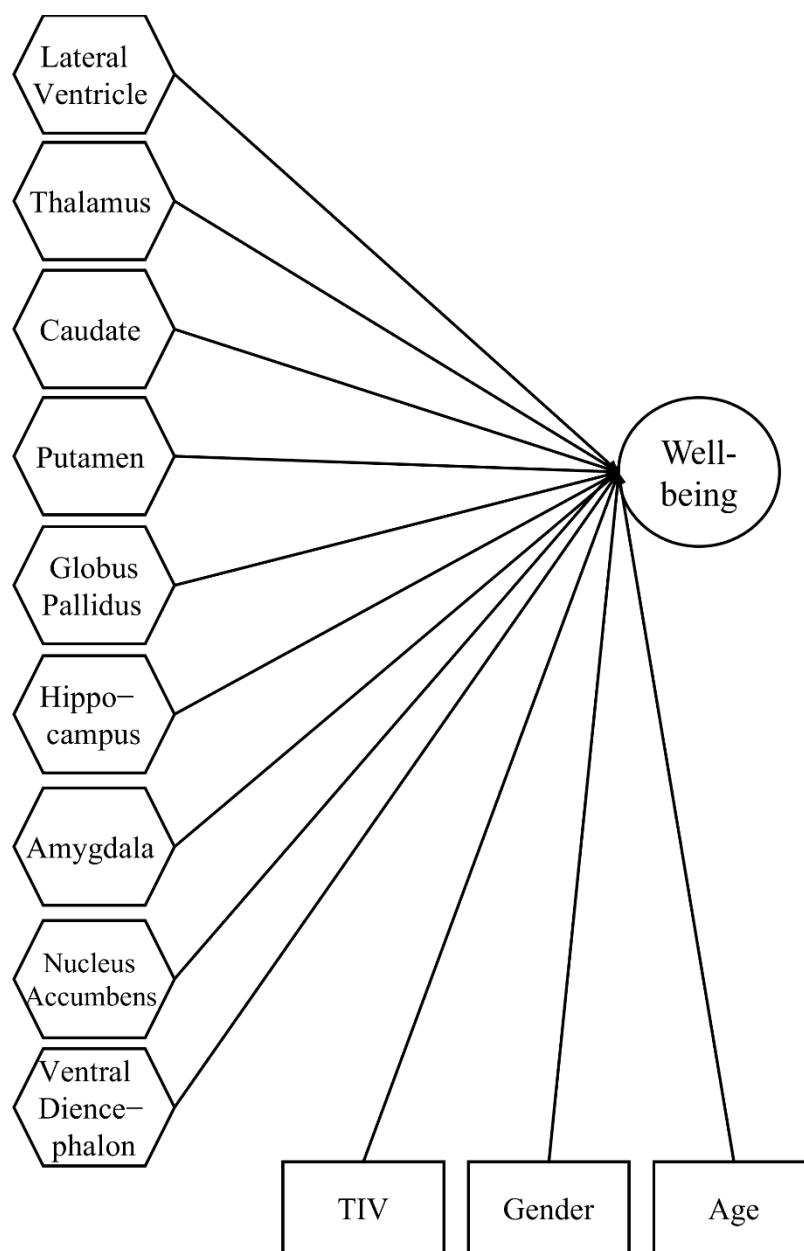


Figure 3. The structural model specified for the HCP data. A circle represents a factor, and hexagons represent components. The error term is omitted.